QS I-GUAGE Certificate for E-Learning Excellence for Academic Digitization

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CONGRATULATIONS

SWAYAM-NPTEL *recognizes*

BASAVESHWAR ENGINEERING COLLEGE

BAGALKOT, KARNATAKA

as a valuable Local Chapter with a rating of

A

based on perfomance in NPTEL Online Certification courses

JAN-APR 2022

PROF. ANDREW THANGARAJ
NPTEL Coordinator
IIT Madras

Rating of NPTEL Local Chapters : AAA - 10; AA - 40; A - 50

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CERTIFICATE

OF APPRECIATION

is awarded to

CHAYALAKSHMI C. L.

for his/her instrumental role as SPOC for the SWAYAM-NPTEL Local Chapter

BASAVESHWAR ENGINEERING COLLEGE

BAGALKOT, KARNATAKA

rated

A

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NPTEL Coordinator
IIT Madras

Sample Digital Content Prepared

Pre-recorded lectures/tutorials sessions

Regular circulation of prerecorded video lectures or tutorial sessions by faculty is made available to the students through remote login system.

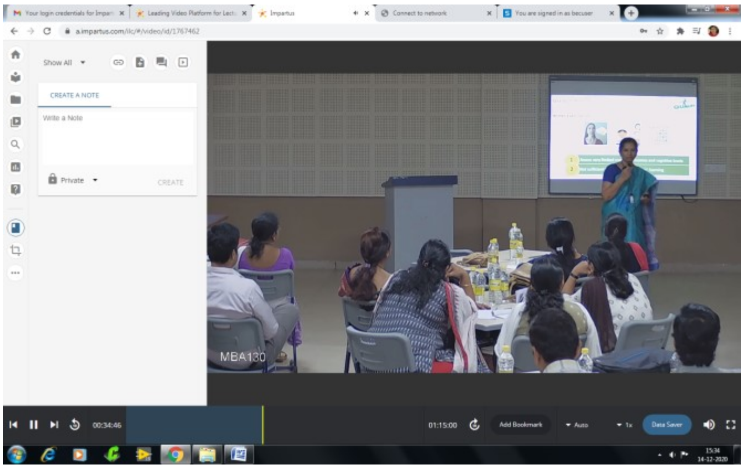
Sample YouTube Video Links

- https://youtu.be/lsutN0o8wRw
- https://youtu.be/XzatwnO_1GU
- https://youtu.be/HB6SQkNBlrE

The College has been effectively utilizing the rich resources available for effective delivery in online mode. These resources are also used for offline teaching process. The important resources are as follows:

1. Lecture Capturing Solution:

- Our institute has procured an interactive lecture capturing platform form “Impartus” that can be used for class room teaching as well as distant teaching learning process.



2. Interactive Smart Board:

Interactive white boards are available in lecture hall and in seminar hall of the department. Faculty makes exclusive use of the facility for effective teaching learning process.

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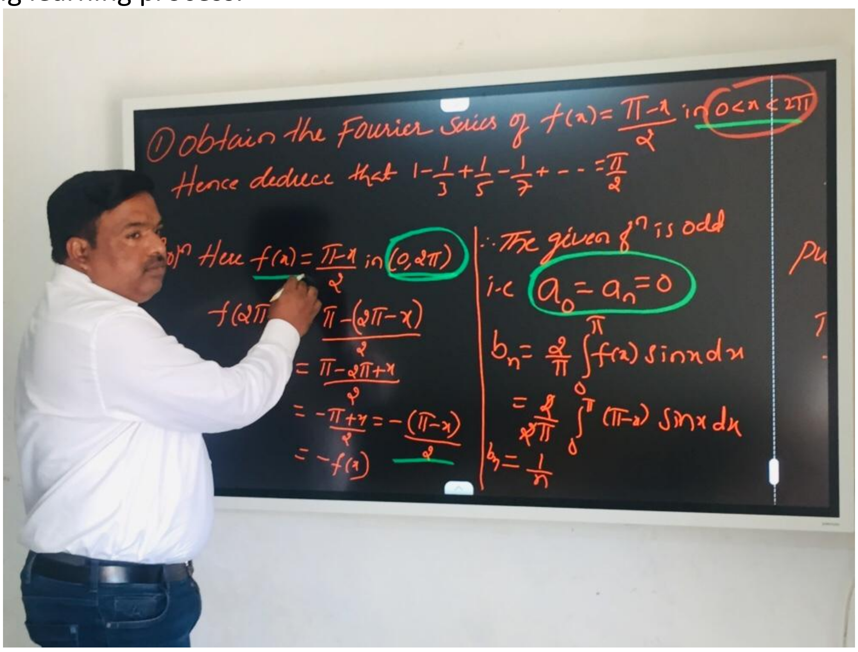
- ### 3. Flip Boards:
- Interactive digital flip board facility is available in lecture hall. Faculty make use of this flip board along with laptop/smart phones to deliver lectures involving animations and simulations. The board has convenient features for active teaching learning process.

- 

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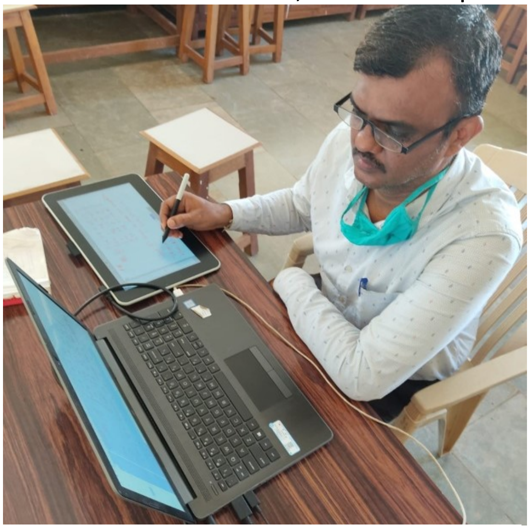
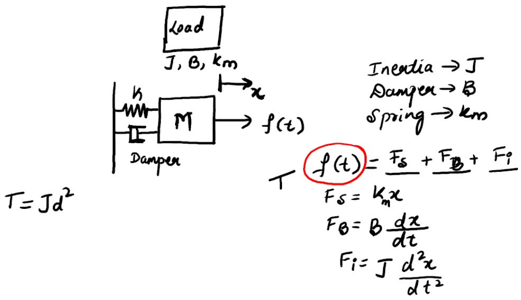
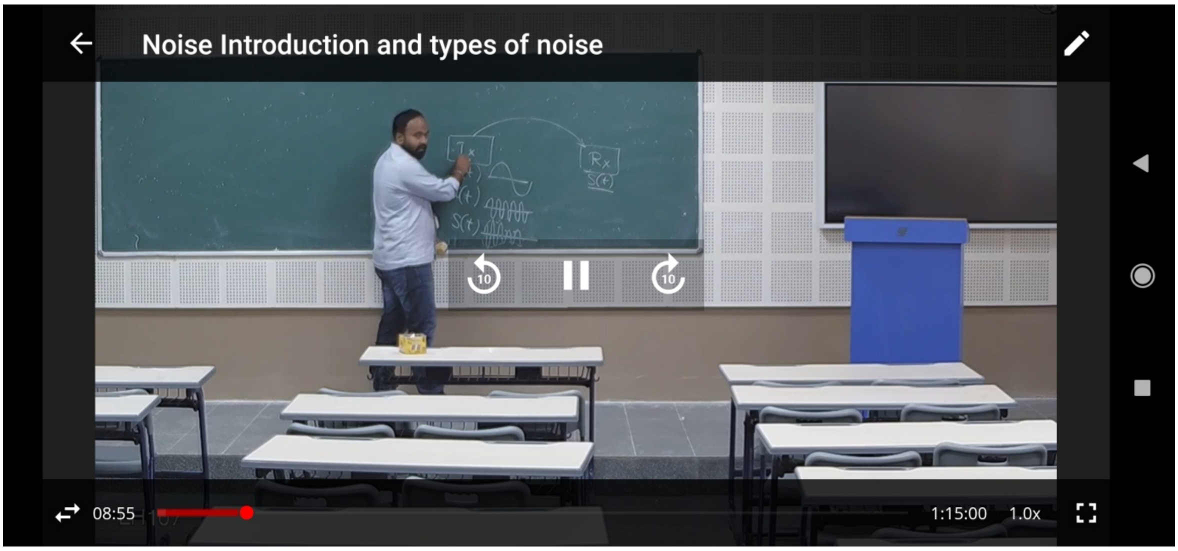
4. Digitalpads

- The institute has procured digital pads which can be exclusively used for online teaching. The pad can be interfaced with laptop/desktop and provides classroom board like environment, which is helpful in interactive teaching learning process.
- 





In another example:

- When a customer makes a purchase online from Dell Computer, the supply chain includes, among others, the customer, Dell's Web site, the Dell assembly plant, and all of Dell's suppliers and their suppliers.
- The Web site provides the customer with information regarding pricing, product variety, and product availability.
- Having made a product choice, the customer enters the order information and pays for the product.
- The customer may later return to the Web site to check the status of the order.
- Stages farther up the supply chain use customer order information to fill the request. That process involves an additional flow of information, product, and funds between various stages of the supply chain.

✓ These examples illustrate that the customer is an integral part of the supply chain.

✓ In fact, the primary purpose of any supply chain is to satisfy customer needs and, in the process, generate profit for itself.

✓ The term *supply chain* conjures up images of product or supply moving from suppliers to manufacturers to distributors to retailers to customers along a chain.

✓ But it is also important to visualize information, funds, and product flows along both directions of this chain.

✓ The term *supply chain* may also imply that only one player is involved at each stage.

In reality, a manufacturer may receive material from several suppliers and then supply several distributors. Thus,

TRANSPORTATION IN A SUPPLY CHAIN

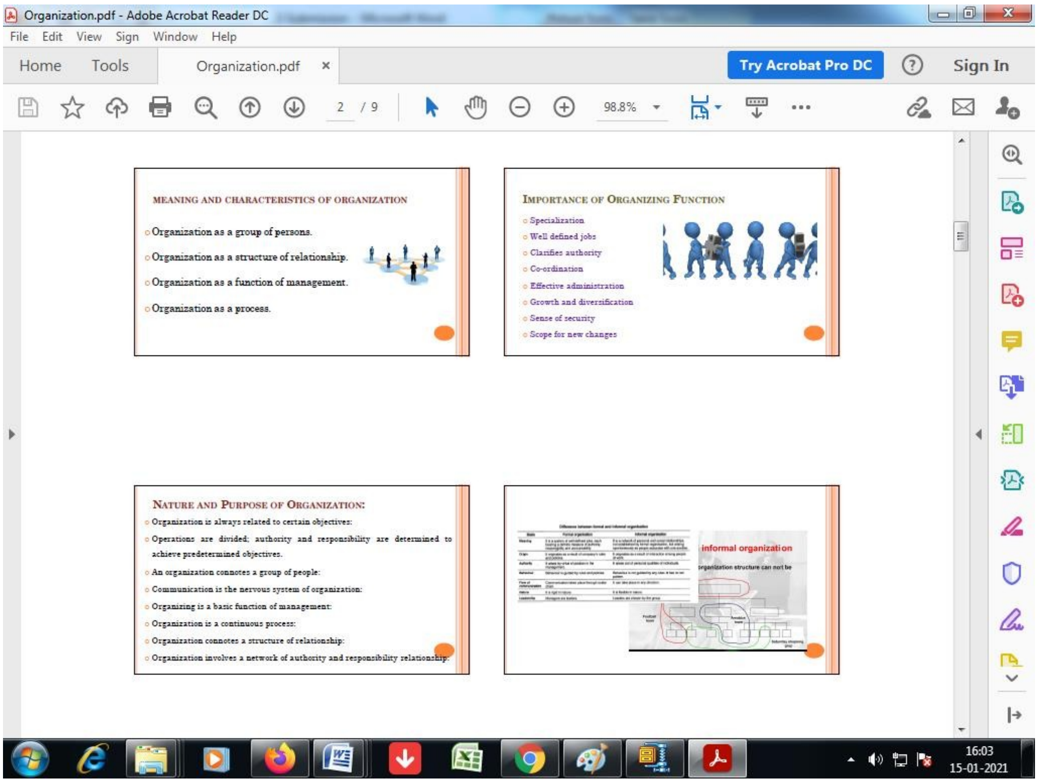
2.2.1. THE ROLE OF TRANSPORTATION IN A SUPPLY CHAIN

Transportation refers to the movement of product from one location to another as it makes its way from the beginning of a supply chain to the customer. Transportation is an important supply chain driver because products are rarely produced and consumed in the same location. Transportation is a significant component of the costs incurred by most supply chains.

The role of transportation is more significant in global supply chains. Dell currently has suppliers worldwide and sells to customers all over the world from just a few plants. Transportation allows products to move across Dell's global network. Dell manufactures in a few locations in the United States and uses responsive transportation provided by package carriers to provide customers with highly customized products at a reasonable price.

International trade is becoming a bigger part of the world's economic activity. Between 1970 and 2001, U.S. international merchandise trade grew by over 20 times, whereas the U.S. economy grew about 10 times. With the rapid growth in international trade, good multimodal freight transportation systems to move the resulting cargo have become even more significant.

Any supply chain's success is closely linked to the appropriate use of transportation. IKEA, the Scandinavian home furnishings retailer, has built a global network with about 180 stores in 23 countries primarily on the basis of effective transportation. IKEA's strategy is built around providing good-quality products at low prices. Their goal is to cut prices by 2 to 3 percent each year. As a result, IKEA works hard to find the most inexpensive global source for each of its products. Modular design of its furniture allows IKEA to transport its goods worldwide much more cost effectively than a traditional furniture manufacturer. The large size of IKEA stores and shipments allows inexpensive


Organization.pdf - Adobe Acrobat Reader DC
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98.8%
MEANING AND CHARACTERISTICS OF ORGANIZATION
Organization as a group of persons.
Organization as a structure of relationship.
Organization as a function of management.
Organization as a process.
IMPORTANCE OF ORGANIZING FUNCTION
Specialization
Well defined jobs
Clarifies authority
Co-ordination
Effective administration
Growth and diversification
Sense of security
Scope for new changes
NATURE AND PURPOSE OF ORGANIZATION:
Organization is always related to certain objectives:
Operations are divided; authority and responsibility are determined to achieve predetermined objectives.
An organization connotes a group of people:
Communication is the nervous system of organization:
Organizing is a basic function of management:
Organization is a continuous process:
Organization connotes a structure of relationship:
Organization involves a network of authority and responsibility relationship:
informal organization
organization structure can not be
16:03
15-01-2021

Design of FIR Filters

(Unit-IV, UEC541C)

Dr. P. N. Kulkarni
Dean (Academic) and Professor in
Dept. of Electronics and Communication Engineering
Basaveshwar Engineering College (Autonomous)
Bagalkot

Design of Digital FIR Filters

- Unlike IIR filters, FIR filters can be designed as the direct approximation to the desired frequency response.

- Because of finite length, FIR filters are inherently stable.

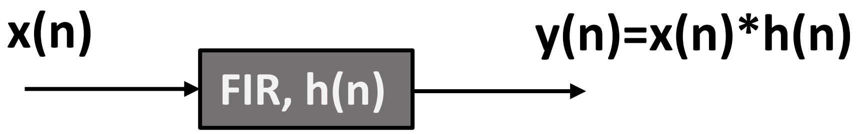
- Phase response is linear, hence are called as constant delay ($\tau=(M-1)/2$) filters.

- Order of the filter is typically large and the filters are mostly non-recursive

- The FIR filters have no poles and hence are all-zero filters

- Generally designed using two methods: (i) windowing method and (ii) frequency sampling technique

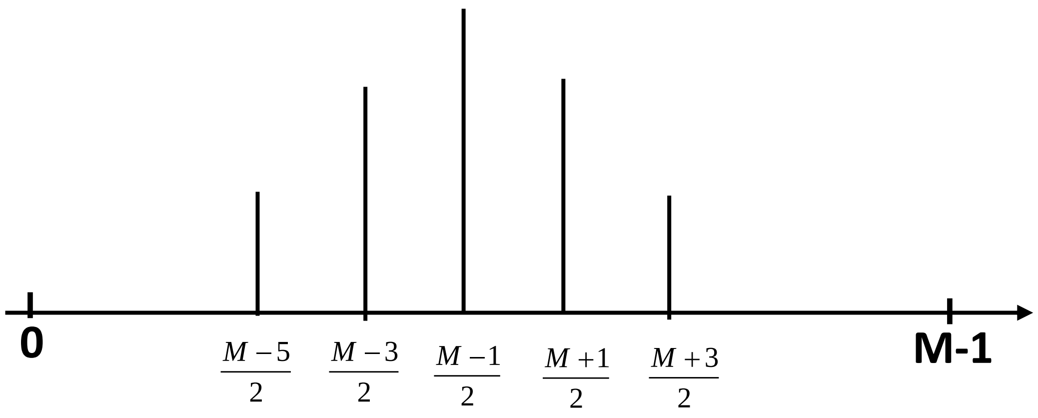
Symmetric and Anti-symmetric Filters



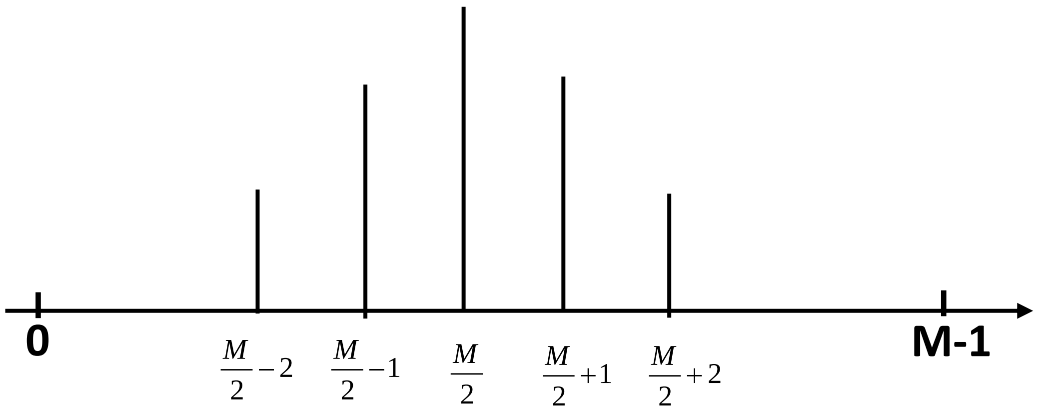
Let h(n) be an impulse response of an FIR filter of length 'M'. The impulse response of an FIR filter is either symmetric or anti-symmetric as shown below

- **If h(n) = h(M-1-n) for all values of n, then it is said to be symmetric**
- **If h(n) = - h(M-1-n) for all values of n, then it is said to be anti-symmetric**

Symmetric filter when n is odd



Symmetric filter when n is even



The transfer function H(z) of an FIR filter can be obtained by taking the z–transform of h(n)

$$\therefore H(z) = \sum_{n=0}^{M-1} h(n)\, z^{-n},$$ for a symmetric FIR filter (assuming M to be odd), we can write

$$H(z) \quad = h(0) + h(1)z^{-1} + h(2)z^{-2} + \ldots\ldots + h\left(\frac{M-3}{2}\right) z^{-\left(\frac{M-3}{2}\right)} + h\left(\frac{M-1}{2}\right) z^{-\left(\frac{M-1}{2}\right)} + h\left(\frac{M+1}{2}\right) z^{-\left(\frac{M+1}{2}\right)} + \ldots\ldots + h(M-1)z^{-(M-1)}$$

$$\therefore H(z) = z^{-\left(\frac{M-1}{2}\right)} \begin{bmatrix} h(0)z^{\left(\frac{M-1}{2}\right)} + h(1)z^{\frac{M-1}{2}-1} + h(2)z^{\frac{M-1}{2}-2} + \ldots\ldots + h\left(\frac{M-3}{2}\right) z^{\frac{M-1}{2}-\left(\frac{M-3}{2}\right)} + h\left(\frac{M-1}{2}\right) \\ + h\left(\frac{M+1}{2}\right) z^{\frac{M-1}{2}-\left(\frac{M+1}{2}\right)} + \ldots\ldots + h(M-1)z^{\frac{M-1}{2}-(M-1)} \end{bmatrix}$$

For symmetric filter, using the relation h(n)=h(M–1–n), we get

$$H(z) = z^{-\left(\frac{M-1}{2}\right)} \left[h(0)\left\{z^{\left(\frac{M-1}{2}\right)} + z^{-\left(\frac{M-1}{2}\right)}\right\}\right] + h(1)\left\{z^{\left(\frac{M-1}{2}-1\right)} + z^{-\left(\frac{M-1}{2}-1\right)}\right\} + \ldots\ldots\ldots + h\left(\frac{M-3}{2}\right)\left\{z^{\left(\frac{M-1}{2}\right)-\frac{M-3}{2}} + z^{-\left(\frac{M-1}{2}-\frac{M-3}{2}\right)}\right\} + h\left(\frac{M-1}{2}\right)$$

$$\therefore H(z) = z^{-\left(\frac{M-1}{2}\right)} \left[h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n)\left\{z^{\left(\frac{M-1}{2}-n\right)} + z^{-\left(\frac{M-1}{2}-n\right)}\right\} \right] \quad \ldots\ldots\ldots\ldots(1)$$

We have equation (1) re−written as follows

$$\therefore H(z) = z^{-\left(\frac{M-1}{2}\right)}\left[h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n)\left(z^{\left(\frac{M-1}{2}-n\right)} + z^{-\left(\frac{M-1}{2}-n\right)}\right)\right] \text{..............}(1)$$

To find the frequency response of an FIR filter, we replace z with e^{jw},

$$\therefore H(z)\Big|_{z=e^{jw}}$$

$$= H\left(e^{jw}\right) = \mathrm{e}^{-jw\left(\frac{M-1}{2}\right)}\left[h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n)\left(e^{jw\left(\frac{M-1}{2}-n\right)} + e^{-jw\left(\frac{M-1}{2}-n\right)}\right)\right]$$

For Symmetric FIR filter, we have,

$$\therefore \left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\text{and } \left|H\left(e^{jw}\right)\right| = 2\sum_{n=0}^{\frac{M}{2}-1} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \qquad \text{when M is even}$$

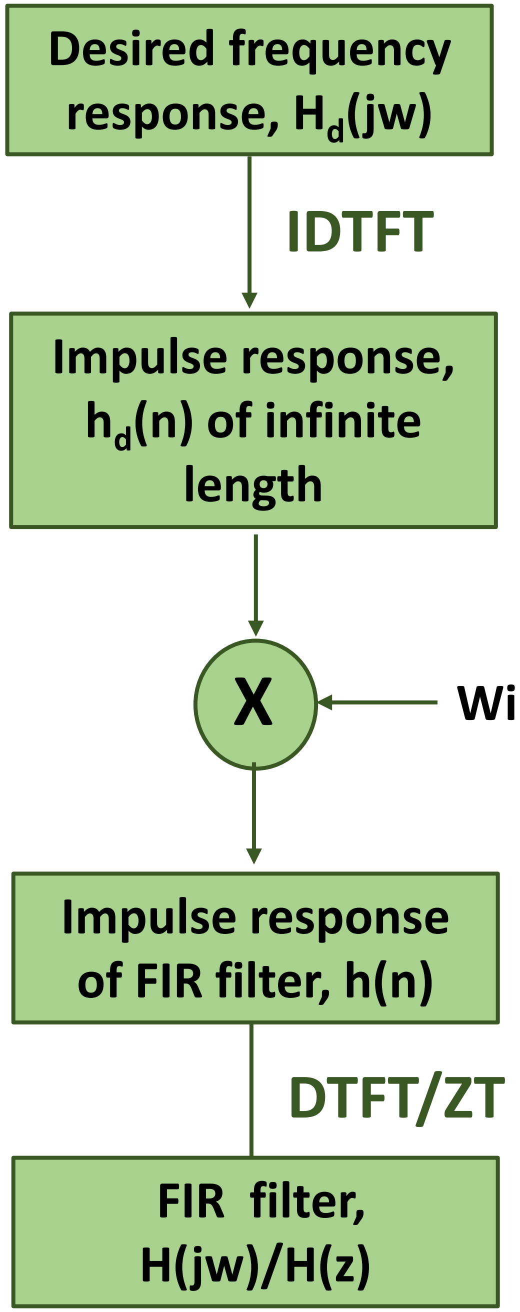
$$\underline{\left|H\left(e^{jw}\right)\right.} = -\omega\left(\frac{M-1}{2}\right)$$

For anti−symmetric FIR filter, we have,

$$\therefore \left|H\left(e^{jw}\right)\right| = 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 2\sum_{n=0}^{\frac{M}{2}-1} h(\mathrm{n})\sin\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is even}$$

Steps involved in designing digital FIR filter using windowing method



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(jw) e^{jwn} dw$$

$$h(n) = h_d(n) \times w(n)$$

$$H(jw) = \sum_{n=0}^{\infty} h(n) e^{-jwn}$$

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

Let $H_d(e^{jw})$ be the desired response (Typically ideal response). Taking IDTFT on both sides, we get

$\mathrm{IDTFT}[H_d(e^{jw})] = \mathrm{h}_d(n) = \frac{1}{2\pi}\int_{2\pi} H_d(e^{jw})e^{jwn}dw$, which gives the impulse response of an IIR filter

Let us define a finite length sequence called window function as follows

$\mathrm{w}(n) = \begin{cases} 1 & 0 \le n \le M-1 \\ 0 & otherwise \end{cases}$ ....(1) this is a rectangular window function

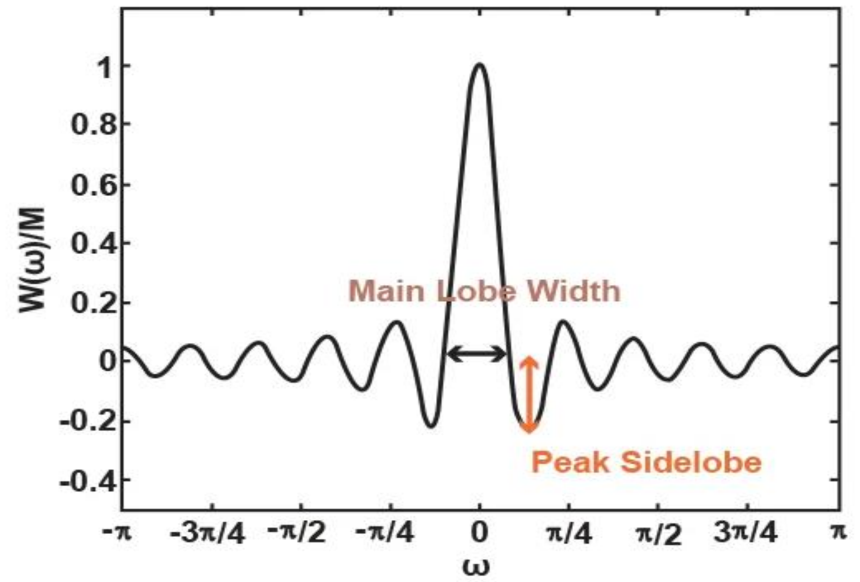
Impulse response of an FIR filter, h(n), is then obtained by truncating an infinite length sequence $h_d(n)$ i.e multiplying $h_d(n)$ with a finite length window function w(n) given by Eqn. (1) as follows

$h(n) = h_d(n) \times w(n)$ ....(2) Taking DTFT on both sides, we get the frquency response of an FIR filter as follows

$H(e^{jw}) = H_d(e^{jw}) * W(e^{jw})$ ....(3) where $H(e^{jw}), H_d(e^{jw})$, and $W(e^{jw})$ are the DTFTs of $h(n)$, $h_d(n)$, and $w(n)$ respectively

$\therefore H(e^{jw}) = \int_{2\pi} H_d(e^{j\theta}) * W(e^{j[w-\theta]})\, d\theta$ ....(4) from convolution integral

From Eqn. (3), it can be seen that $H(e^{jw}) \to H_d(e^{jw})$ when $W(e^{jw}) \to \delta(w) \Rightarrow w(n)$ must be constant for all values of $'\mathrm{n}'$ which is possible only when w(n) is of infinite length (i.e no windowing at all!). Thus for all windw sequence of finite length the obtained response $H(e^{jw})$ differs from $H_d(e^{jw})$. However, judicious selection of window length can reduce the difference.



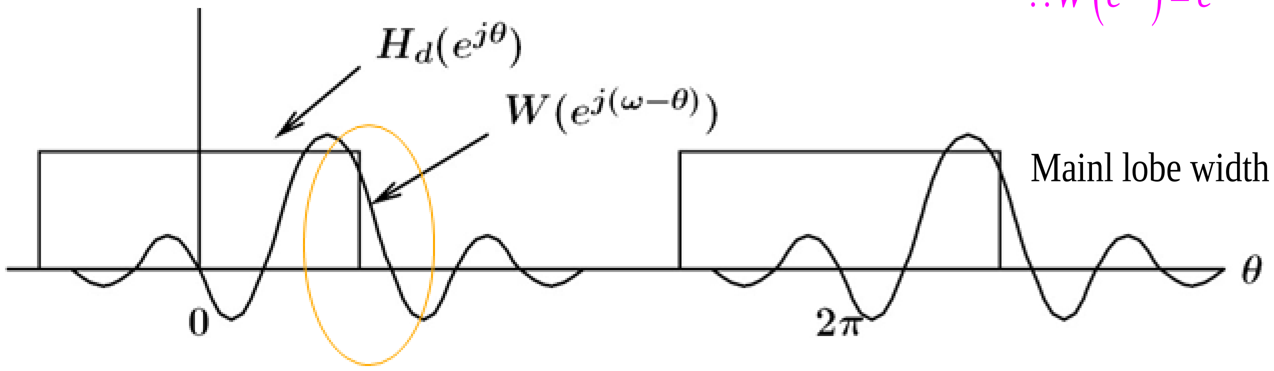
We have rectangular window defined as: $w(n)=\begin{cases}1 & 0\le n\le M-1\\ 0 & otherwise\end{cases}$

$$DTFT\left(w(n)\right)=W\left(e^{jw}\right)=\sum_{n=0}^{M-1}1\times e^{-jwn}=\sum_{n=0}^{M-1}\left(e^{-jw}\right)^{n}$$

$$\therefore W\left(e^{jw}\right)=\frac{1-e^{-jwM}}{1-e^{-jw}}=\frac{e^{-j\frac{wM}{2}}\left(e^{j\frac{wM}{2}}-e^{-j\frac{wM}{2}}\right)}{e^{-j\frac{w}{2}}\left(e^{j\frac{w}{2}}-e^{-j\frac{w}{2}}\right)}$$, Dividing both numerator and denominator by 2j, we get

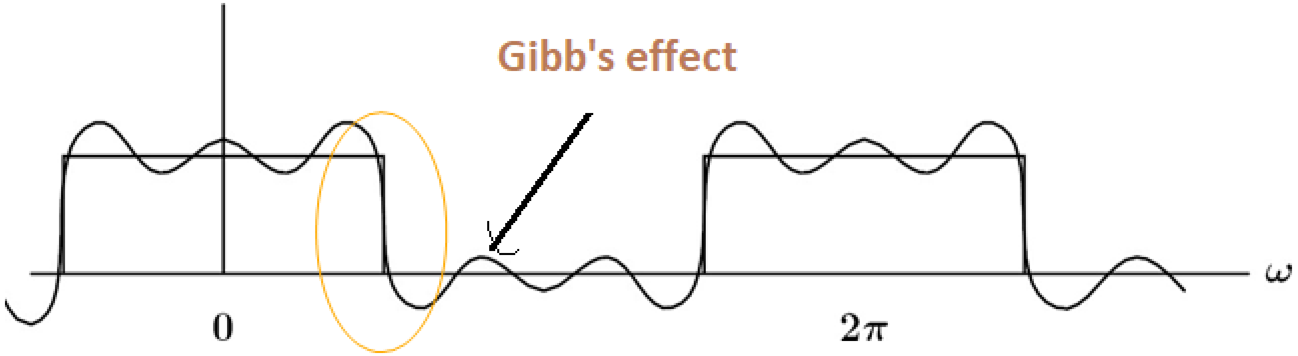
$$\therefore W\left(e^{jw}\right)=e^{-j\frac{w}{2}(M-1)}\frac{\sin\left(\frac{wM}{2}\right)}{\sin\left(\frac{w}{2}\right)}$$ The zero crossing occurs at $w=\frac{2k\pi}{M}$

Mainl lobe width $=\frac{4\pi}{M}$



During the evaluation of $H\left(e^{jw}\right)=\int_{2\pi}H_d\left(e^{j\theta}\right)*W\left(e^{j[w-\theta]}\right)d\theta$

whenever $W\left(e^{j[w-\theta]}\right)$ glides throgh the point of abrupt discontinuity,
oscillations with non-uniform convergance are formed
these distortion caused due to non-uniformly converging
oscillations is reffered to as Gibb's effect.



Ideally we would like to have

M small – few computations

$W(\omega)$ - close to a delta function (which implies 'M' to be large) for the FIR filter response to be close the ideal response

$$H_d(\omega) = \begin{cases} 1 \text{ if } |\omega| \le \omega_c \\ 0 \text{ if } \omega_c < |\omega| < \pi \end{cases}$$

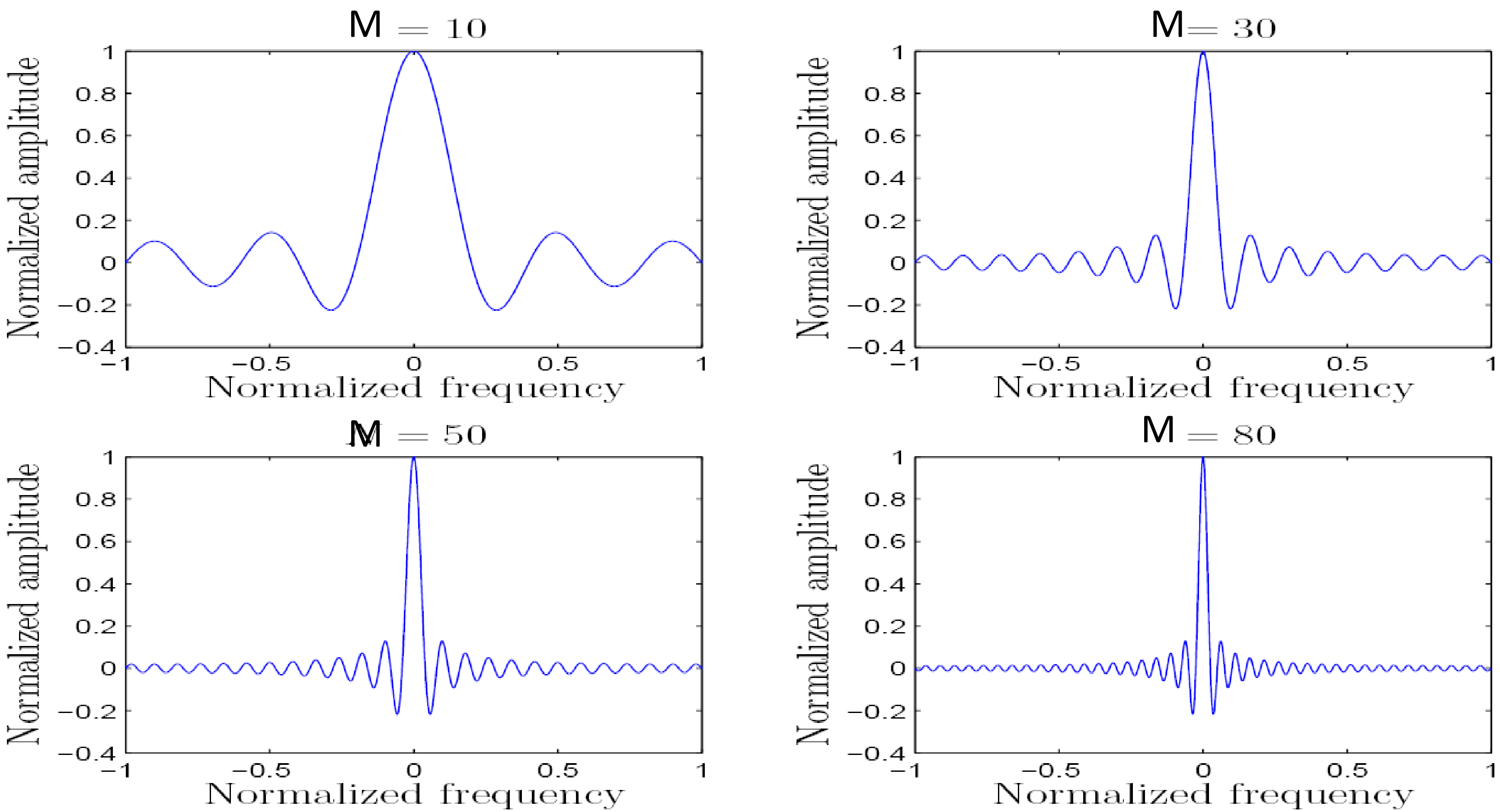
our ideal low-pass filter

These two requirements are conflicting!

Characteristics of Rectangular window

1. **Width of main lobe, 4л/M, decreases with increase in M**
2. **Magnitude of side lobes are high and independent of M**
3. **Width of side lobes decreases with increase in M such that the area under the side lobes remain constant**
4. **Abrupt discontinuity in time domain result in large side lobes leading to undesired ringing effect in the FIR filters**

Effect of increase in the length of window function



- The width of the main lobe decreases as M increases
- The area under sidelobes remain constant as M increases

Commonly used Window functions for FIR filter design

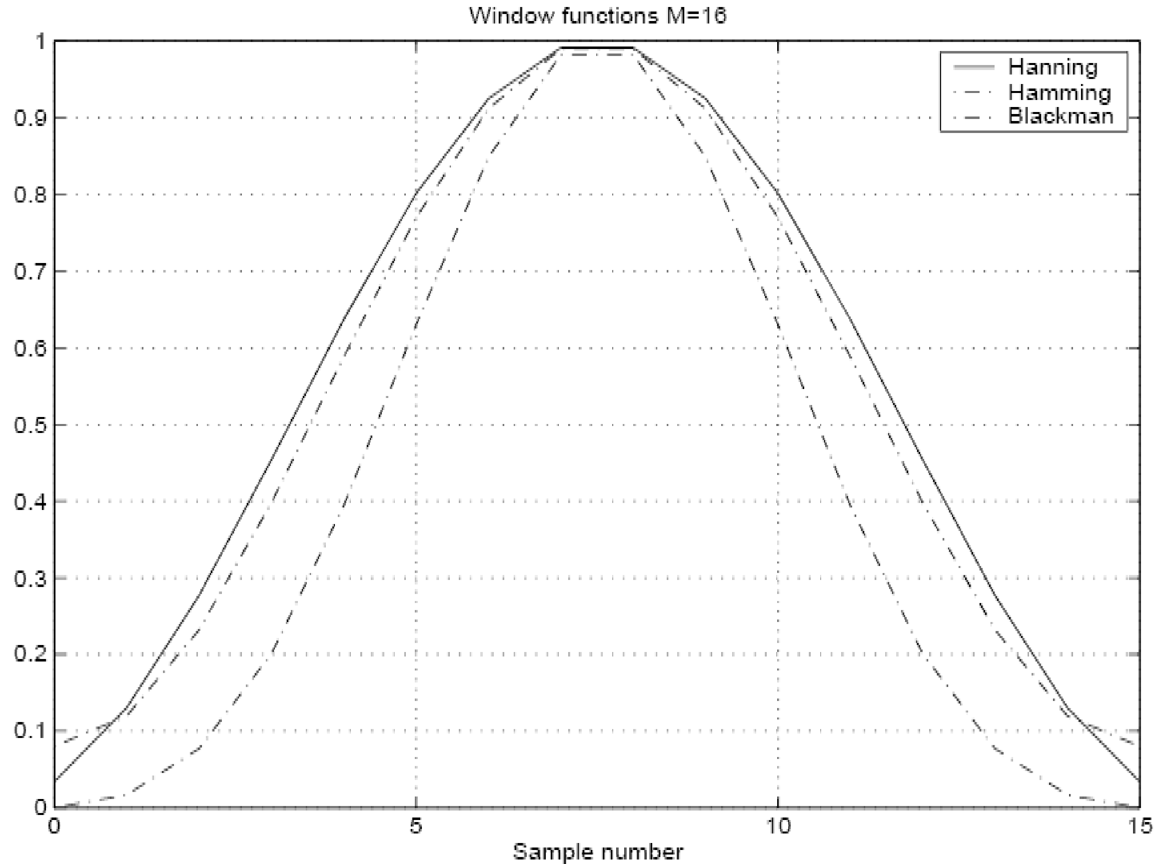
1. Bartlet (Triangular) window: $w(n) = 1 - \dfrac{2\left|n - \dfrac{M-1}{2}\right|}{M-1}$

2. Blackman window : $w(n) = 0.42 - 0.5\cos\dfrac{2\pi n}{M-1} + 0.08\cos\dfrac{4\pi n}{M-1}$

3. Hamming window : $w(n) = 0.54 - 0.46\cos\dfrac{2\pi n}{M-1}$

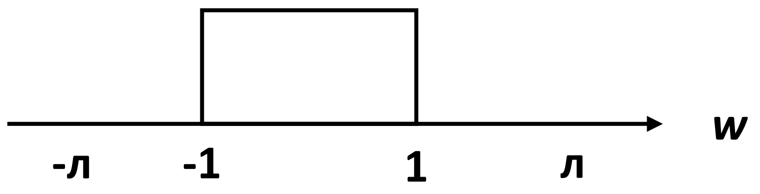
4. Hanning window : $w(n) = \dfrac{1}{2}\left[1 - \cos\dfrac{2\pi n}{M-1}\right]$

5. Rectangular window : $w(n) = \begin{cases} 1 & 0 \le n \le M-1 \\ 0 & otherwise \end{cases}$



Example 1: Design an FIR low pass filter with a constant delay of 3 samples using different types of window functions. The desired frequency response is given below

$$H_d(e^{jw}) = \begin{cases} e^{-j\tau w} & -1 \le |w| \le 1 \\ 0 & otherwise \end{cases} = \begin{cases} e^{-j3w} & -1 \le |w| \le 1 \\ 0 & otherwise \end{cases}$$



Solution

$$\tau = \frac{M-1}{2} = 3 \Rightarrow M = 7 \text{ is the length of FIR filter}$$

$$\text{IDTFT}\left[H_d\left(e^{jw}\right)\right] = h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\left(e^{jw}\right)e^{jwn}dw = \frac{1}{2\pi}\int_{-1}^{1} e^{-j3w}e^{jwn}dw = \frac{1}{2\pi}\int_{-1}^{1} e^{jw(n-3)}dw$$

$$\therefore h_d(n) = \frac{1}{2\pi}\left[\frac{e^{jw(n-3)}}{j(n-3)}\right]_{-1}^{1} = \frac{1}{2\pi}\left[\frac{e^{j(n-3)} - e^{-j(n-3)}}{j(n-3)}\right]$$

$$\therefore h_d(n) = \frac{1}{\pi}\left[\frac{\sin(n-3)}{(n-3)}\right]$$

$$\therefore h_d(n) = \begin{cases} \frac{1}{\pi}\left[\frac{\sin(n-3)}{(n-3)}\right] & n \ne 3 \\ \frac{1}{\pi} & n = 3 \end{cases}$$

We have $h_d(n) = \begin{cases} \frac{1}{\pi}\left[\frac{\sin(n-3)}{(n-3)}\right] & n \neq 3 \\ \frac{1}{\pi} & n = 3 \end{cases}$ since the filter is Symmetric i. e. $h_d(n) = h_d(M-1-n)$

$\therefore h_d(0) = h_d(6) = 0.0149$

$h_d(1) = h_d(5) = 0.1447$

$h_d(2) = h_d(4) = 0.2678$

$h_d(3) = \frac{1}{\pi} = 0.31831$

(1) Rectangular window: $w(n) = 1 \quad$ for $0 \leq \text{n} \leq 6$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n) = h_d(n) \quad \because w(n) = 1$

$\therefore h(0) = h(6) = 0.0149$

$h(1) = h(5) = 0.1447$

$h(2) = h(4) = 0.2678$

$h(3) = \frac{1}{\pi} = 0.31831$

For Symmetric FIR filter with odd value of M, the magnitude frequency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2}-n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.31831 + 2\left[0.0149\cos(3w) + 0.1447\cos(2w) + 0.2678\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.31831 + 0.0298\cos(3w) + 0.2894\cos(2w) + 0.5356\cos(w)$$

(2) Hanning Window: $w(n) = \frac{1}{2}\left[1-\cos\frac{2\pi n}{M-1}\right] = \frac{1}{2}\left[1-\cos\frac{2\pi n}{6}\right] = \frac{1}{2}\left[1-\cos\frac{\pi n}{3}\right]$ for n=0, 1, 2.....6

The Hanning window values are given by

$$\therefore w(0) = w(6) = 0$$

$$w(1) = w(5) = \frac{1}{4}$$

$$w(2) = w(4) = \frac{3}{4}$$

$$w(3) = 1$$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n)$

$$\therefore h(0) = h(6) = h_d(0) \times w(0) = 0$$

$$h(1) = h(5) = h_d(1) \times w(1) = 0.1447 \times \frac{1}{4} = 0.03618$$

$$h(2) = h(4) = h_d(2) \times w(2) = 0.26785 \times \frac{3}{4} = 0.20089$$

$$h(3) = 0.31831 \times 1 = 0.31831$$

For Symmetric FIR filter with odd value of M, the manitude frquency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.31831 + 2\left[0.03618\cos(2w) + 0.20089\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.31831 + 0.07236\cos(2w) + 0.40178\cos(w)$$

(3) Hamming Window: $w(n)=0.54-0.46\cos\frac{2\pi n}{M-1}=0.54-0.46\cos\frac{2\pi n}{6}$ for n=0, 1, 2.....6

The Hamming window values are given by

$\therefore w(0)=w(6)=0.08$

$w(1)=w(5)=0.31$

$w(2)=w(4)=0.77$

$w(3)=1$

Impulse response of FIR filter is given by: $h(n)=h_d(n)\times w(n)$

$\therefore h(0)=h(6)=h_d(0)\times w(0)=0.0149\times 0.08=1.192\times 10^{-3}$

$h(1)=h(5)=h_d(1)\times w(1)=0.1447\times 0.31=0.0448$

$h(2)=h(4)=h_d(2)\times w(2)=0.26785\times 0.77=0.2062$

$h(3)=0.31831\times 1=0.31831$

For Symmetric FIR filter with odd value of M, the manitude frquency response is given by,

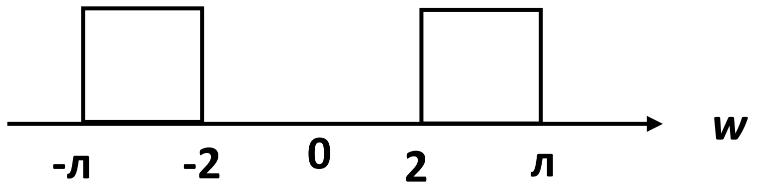
$$\left|H\left(e^{jw}\right)\right|=h\left(\frac{M-1}{2}\right)+2\sum_{n=0}^{\frac{M-3}{2}}h(n)\cos\omega\left(\frac{M-1}{2}-n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right|=h(3)+2\sum_{n=0}^{2}h(n)\cos\omega(3-n)=0.31831+2\left[1.192\times 10^{-3}\cos(3w)+0.0448\cos(2w)+0.2062\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right|=0.31831+0.0024\cos(3w)+0.08973\cos(2w)+0.41249\cos(w)$$

Example 2: Design an FIR high pass filter with a constant delay of 3 samples using different types of window functions. The desired frequency response is given below

$$H_d(e^{jw}) = \begin{cases} e^{-j\tau w} & 2 \le |w| \le \pi \\ 0 & otherwise \end{cases} = \begin{cases} e^{-j3w} & 2 \le |w| \le \pi \\ 0 & otherwise \end{cases}$$



Solution

$$\tau = \frac{M-1}{2} = 3 \Rightarrow M = 7 \text{ is the length of FIR filter}$$

$$\text{IDTFT}\left[H_d\left(e^{jw}\right)\right] = h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d\left(e^{jw}\right)e^{jwn}dw = \frac{1}{2\pi}\left[\int_{-\pi}^{-2} e^{-j3w}e^{jwn}dw + \int_{2}^{\pi} e^{-j3w}e^{jwn}dw\right] = \frac{1}{2\pi}\left[\int_{-\pi}^{-2} e^{j(n-3)w}dw + \int_{2}^{\pi} e^{j(n-3)w}dw\right]$$

$$\therefore h_d(n) = \frac{1}{2\pi}\left[\frac{e^{jw(n-3)}}{j(n-3)}\right]_{-\pi}^{-2} + \frac{1}{2\pi}\left[\frac{e^{jw(n-3)}}{j(n-3)}\right]_{2}^{\pi} = \frac{1}{2\pi}\left[\frac{e^{-j2(n-3)} - e^{-j\pi(n-3)} + e^{j\pi(n-3)} - e^{j2(n-3)}}{j(n-3)}\right]$$

$$\therefore h_d(n) = \frac{1}{\pi}\left[\frac{\sin\pi(n-3) - \sin 2(n-3)}{(n-3)}\right]$$

$$\therefore h_d(n) = \begin{cases} \frac{1}{\pi}\left[\frac{\sin\pi(n-3) - \sin 2(n-3)}{(n-3)}\right] & n \ne 3 \\ 1 - \frac{2}{\pi} = 0.36338 & n = 3 \end{cases}$$

We have $h_d(n) = \begin{cases} \dfrac{1}{\pi}\left[\dfrac{\sin\pi(n-3) - \sin 2(n-3)}{(n-3)}\right] & n \neq 3 \\ 1 - \dfrac{2}{\pi} = 0.36338 & n = 3 \end{cases}$

since the filter is Symmetric i. e. $h_d(n) = h_d(M-1-n)$

$\therefore h_d(0) = h_d(6) = 0.02965$

$h_d(1) = h_d(5) = 0.12045$

$h_d(2) = h_d(4) = -0.28944$

$h_d(3) = 1 - \dfrac{2}{\pi} = 0.36338$

(1) Rectangular window: $w(n) = 1$ for $0 \leq n \leq 6$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n) = h_d(n) \quad \because w(n) = 1$

$\therefore h(0) = h(6) = 0.02965$

$h(1) = h(5) = 0.12045$

$h(2) = h(4) = -0.28944$

$h(3) = 0.36338$

For Symmetric FIR filter with odd value of M, the magnitude frequency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2}-n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.36338 + 2\left[0.02965\cos(3w) + 0.12045\cos(2w) - 0.28944\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.36338 + 0.0593\cos(3w) + 0.2409\cos(2w) - 0.57888\cos(w)$$

(2) Hanning Window: $w(n) = \frac{1}{2}\left[1-\cos\frac{2\pi n}{M-1}\right] = \frac{1}{2}\left[1-\cos\frac{2\pi n}{6}\right] = \frac{1}{2}\left[1-\cos\frac{\pi n}{3}\right]$ for n=0, 1, 2.....6

The Hanning window values are given by

$$\therefore w(0) = w(6) = 0$$

$$w(1) = w(5) = \frac{1}{4}$$

$$w(2) = w(4) = \frac{3}{4}$$

$$w(3) = 1$$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n)$

$$\therefore h(0) = h(6) = h_d(0) \times w(0) = 0.02965 \times 0 = 0$$

$$h(1) = h(5) = h_d(1) \times w(1) = 0.12045 \times \frac{1}{4} = 0.0301$$

$$h(2) = h(4) = h_d(2) \times w(2) = -0.28944 \times \frac{3}{4} = 0.21708$$

$$h(3) = 0.36338 \times 1 = 0.36338$$

For Symmetric FIR filter with odd value of M, the manitude frquency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.36338 + 2\left[0.0301\cos(2w) - 0.21708\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.36338 + 0.0602\cos(2w) - 0.43146\cos(w)$$

(3) Hamming Window: $w(n) = 0.54 - 0.46\cos\frac{2\pi n}{M-1} = 0.54 - 0.46\cos\frac{2\pi n}{6}$ for n=0, 1, 2.....6

The Hamming window values are given by

$\therefore w(0) = w(6) = 0.08$

$w(1) = w(5) = 0.31$

$w(2) = w(4) = 0.77$

$w(3) = 1$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n)$

$\therefore h(0) = h(6) = h_d(0) \times w(0) = 0.02965 \times 0.08 = 0.00237$

$h(1) = h(5) = h_d(1) \times w(1) = 0.12045 \times 0.31 = 0.0373$

$h(2) = h(4) = h_d(2) \times w(2) = -0.28944 \times 0.77 = 0.2228$

$h(3) = 0.36338 \times 1 = 0.36338$

For Symmetric FIR filter with odd value of M, the manitude frquency response is given by,

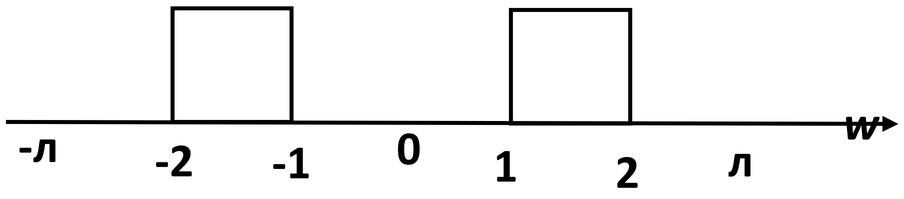
$$|H(e^{jw})| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore |H(e^{jw})| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega\,(3-n) = 0.36338 + 2[0.00237\cos(3w) + 0.03733\cos(2w) - 0.222\cos(w)]$$

$$\therefore |H(e^{jw})| = 0.36338 + 0.00474\cos(3w) + 0.07468\cos(2w) - 0.444\cos(w)$$

Example 3: Design an FIR Band pass filter with a constant delay of 3 samples using different types of window functions. The desired frequency response is given below

$$H_d(e^{jw}) = \begin{cases} e^{-j\tau w} & 1 \le |w| \le 2 \\ 0 & otherwise \end{cases} = \begin{cases} e^{-j3w} & 1 \le |w| \le 2 \\ 0 & otherwise \end{cases}$$



Solution

$$\tau = \frac{M-1}{2} = 3 \Rightarrow M = 7 \text{ is the length of FIR filter}$$

$$\text{IDTFT}[H_d(e^{jw})] = h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw})\, e^{jwn} dw = \frac{1}{2\pi}\left[\int_{-2}^{-1} e^{-j3w} e^{jwn}\, dw + \int_{1}^{2} e^{-j3w} e^{jwn}\, dw\right]$$

$$= \frac{1}{2\pi}\left[\int_{-2}^{1} e^{j(n-3)w}\, dw + \int_{1}^{2} e^{j(n-3)w}\, dw\right]$$

$$\therefore h_d(n) = \frac{1}{2\pi}\left[\frac{e^{jw(n-3)}}{j(n-3)}\right]_{-2}^{-1} + \frac{1}{2\pi}\left[\frac{e^{jw(n-3)}}{j(n-3)}\right]_{1}^{2} = \frac{1}{2\pi}\left[\frac{e^{-j(n-3)} - e^{-j2(n-3)} + e^{j2(n-3)} - e^{j(n-3)}}{j(n-3)}\right]$$

$$\therefore h_d(n) = \frac{1}{\pi}\left[\frac{-\sin\pi\,(n-3) + \sin 2\,(n-3)}{(n-3)}\right]$$

$$\therefore h_d(n) = \begin{cases} \frac{1}{\pi}\left[\frac{\sin 2\,(n-3) - \sin(n-3)}{(n-3)}\right] & n \ne 3 \\ \frac{1}{\pi} = 0.3183 & n = 3 \end{cases}$$

We have $h_d(n) = \begin{cases} \frac{1}{\pi}\left[\frac{\sin 2(n-3) - \sin(n-3)}{(n-3)}\right] & n \neq 3 \\ \frac{1}{\pi} = 0.3183 & n = 3 \end{cases}$

since the filter is Symmetric i. e. $h_d(n) = h_d(M-1-n)$

$\therefore h_d(0) = h_d(6) = -0.04462$

$h_d(1) = h_d(5) = -0.26517$

$h_d(2) = h_d(4) = 0.02159$

$h_d(3) = \frac{1}{\pi} = 0.31831$

(1) Rectangular window: $w(n) = 1$ for $0 \leq n \leq 6$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n) = h_d(n) \quad \because w(n) = 1$

$\therefore h(0) = h(6) = -0.04462$

$h(1) = h(5) = -0.26517$

$h(2) = h(4) = 0.02159$

$h(3) = 0.31831$

For Symmetric FIR filter with odd value of M, the magnitude frequency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.31831 + 2\left[-0.04462\cos(3w) - 0.26517\cos(2w) + 0.02159\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.31831 - 0.08924\cos(3w) - 0.53034\cos(2w) + 0.04318\cos(w)$$

(2) Hanning Window: $w(n) = \frac{1}{2}\left[1 - \cos\frac{2\pi n}{M-1}\right] = \frac{1}{2}\left[1 - \cos\frac{2\pi n}{6}\right] = \frac{1}{2}\left[1 - \cos\frac{\pi n}{3}\right]$ for n=0, 1, 2.....6

The Hanning window values are given by

$$\therefore w(0) = w(6) = 0$$

$$w(1) = w(5) = \frac{1}{4}$$

$$w(2) = w(4) = \frac{3}{4}$$

$$w(3) = 1$$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n)$

$$\therefore h(0) = h(6) = h_d(0) \times w(0) = -0.04462 \times 0 = 0$$

$$h(1) = h(5) = h_d(1) \times w(1) = -0.26517 \times \frac{1}{4} = -0.0663$$

$$h(2) = h(4) = h_d(2) \times w(2) = 0.02159 \times \frac{3}{4} = 0.01619$$

$$h(3) = 0.31831 \times 1 = 0.36338$$

For Symmetric FIR filter with odd value of M, the manitude frquency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.31831 + 2\left[-0.0663\cos(2w) + 0.01619\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.31831 - 0.1326\cos(2w) + 0.03238\cos(w)$$

(3) Hamming Window: $w(n) = 0.54 - 0.46\cos\frac{2\pi n}{M-1} = 0.54 - 0.46\cos\frac{2\pi n}{6}$ for n=0, 1, 2.....6

The Hamming window values are given by

$\therefore w(0) = w(6) = 0.08$

$w(1) = w(5) = 0.31$

$w(2) = w(4) = 0.77$

$w(3) = 1$

Impulse response of FIR filter is given by: $h(n) = h_d(n) \times w(n)$

$\therefore h(0) = h(6) = h_d(0) \times w(0) = -0.04462 \times 0.08 = -0.0357$

$h(1) = h(5) = h_d(1) \times w(1) = -0.26517 \times 0.31 = -0.0822$

$h(2) = h(4) = h_d(2) \times w(2) = 0.02159 \times 0.77 = 0.0166$

$h(3) = 0.31831 \times 1 = 0.31831$

For Symmetric FIR filter with odd value of M, the manitude frquency response is given by,

$$\left|H\left(e^{jw}\right)\right| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2} - n\right) \quad \text{when M is odd}$$

$$\therefore \left|H\left(e^{jw}\right)\right| = h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(3-n) = 0.31831 + 2\left[-0.0357\cos(3w) - 0.0822\cos(2w) + 0.0166\cos(w)\right]$$

$$\therefore \left|H\left(e^{jw}\right)\right| = 0.31831 - 0.0714\cos(3w) - 0.1644\cos(2w) + 0.0322\cos(w)$$

Characteristics of window functions

Window Type	Side lobe amplitude (dB)	Transition width	Stop band Gain (dB)	Mainlobe width
Rectangular	-13	0.9/N	-21	4л/M
Hanning	-31	3.1/N	-44	8л/M
Hamming	-41	3.3/N	-53	8л/M
Blackan	-57	5.5/N	-74	12л/M

Frequency Sampling Technique of FIR Filter Design

Let h(n) be the impulse response of an FIR filter of length N

$$\therefore H(z)=\sum_{n=0}^{N-1} h(n) z^{-n}, \text{ since } \mathrm{h}(n)=IDFT\left[H(k)\right]=\frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{j\frac{2\pi}{N}kn}$$

$$H(z)=\sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{j\frac{2\pi}{N}kn}\right] z^{-n}$$

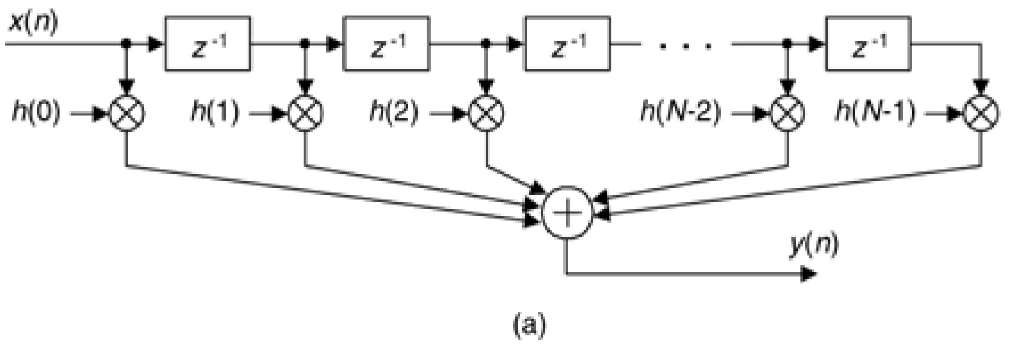
$$=\frac{1}{N}\sum_{k=0}^{N-1} H(k)\sum_{n=0}^{N-1}\left(e^{j\frac{2\pi}{N}k} z^{-1}\right)^{n}$$

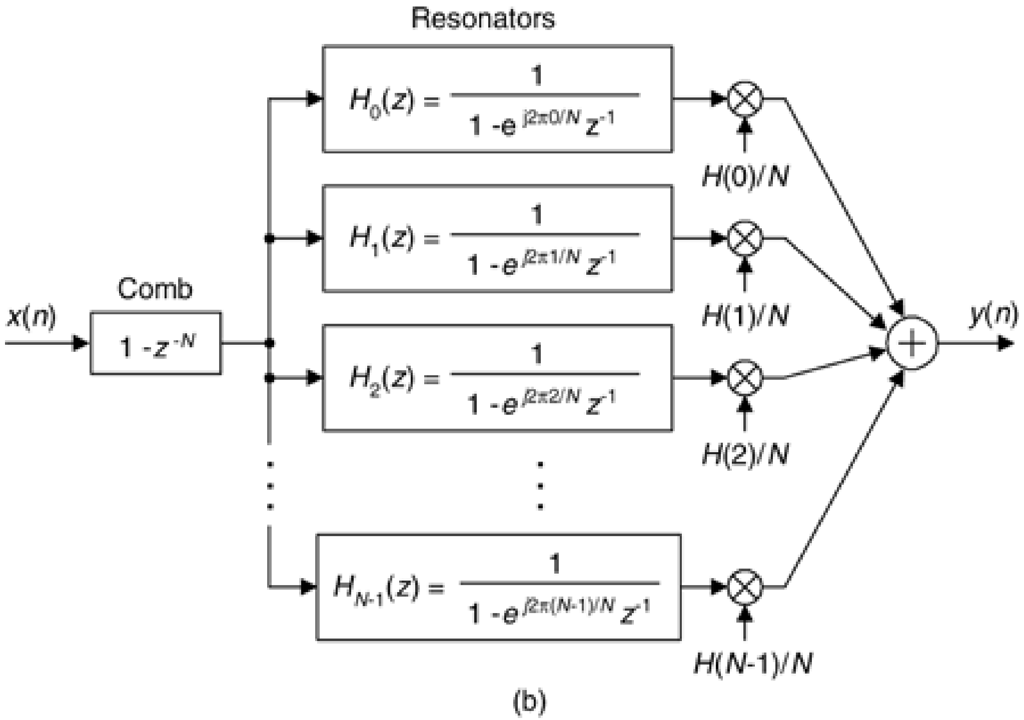
$$=\frac{1}{N}\sum_{k=0}^{N-1} H(k)\frac{1-e^{j\frac{2\pi}{N}kN} z^{-N}}{1-e^{j\frac{2\pi}{N}k} z^{-1}}=\frac{1}{N}\sum_{k=0}^{N-1} H(k)\frac{1-z^{-N}}{1-e^{j\frac{2\pi}{N}k} z^{-1}}$$

$$\therefore H(z)=\left[1-z^{-N}\right].\left[\frac{1}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-e^{j\frac{2\pi}{N}k} z^{-1}}\right]$$

Where $\left[1-z^{-N}\right]$ is a comb filter, and

$$H_k(z)=\frac{1}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-e^{j\frac{2\pi}{N}k} z^{-1}}$$ is a single pole resonator





Using Conjugate symmetric property of DFT, we have
H(N-K) = H*(k),   for k = 0, 1, 2,........(N-1)/2 when N is odd and
H(N-K) = H*(k),   for k = 0, 1, 2,........(N/2)-1 & H(N/2) = 0 when N is even

$$\text{We have } h(n) = IDFT[H(k)] = \frac{1}{N}\sum_{k=0}^{N-1} H(k)\, e^{j\frac{2\pi}{N}kn}$$

$$\therefore h(n) = \frac{1}{N}\left[\begin{matrix} H(0) + \underline{H(1)e^{j\frac{2\pi}{N}1.n} + H(N-1)e^{j\frac{2\pi}{N}(N-1)n}} + \underline{H(2)e^{j\frac{2\pi}{N}1.n} + H(N-2)e^{j\frac{2\pi}{N}(N-2)n}} + \ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots + \underline{H\left(\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n} + H\left(N-\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(N-\frac{N-1}{2}\right).n}} \end{matrix}\right]$$

$$\therefore h(n) = \frac{1}{N}\left[\begin{matrix} H(0) + \underline{H(1)e^{j\frac{2\pi}{N}1.n} + H(1)^{*}e^{-j\frac{2\pi}{N}n}} + \underline{H(2)e^{j\frac{2\pi}{N}2.n} + H(2)^{*}e^{-j\frac{2\pi}{N}2n}} + \ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots + \underline{H\left(\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n} + H\left(\frac{N-1}{2}\right)^{*}e^{-j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n}} \end{matrix}\right]$$

We have $h(n)=\frac{1}{N}\left[\begin{array}{l} H(0)+\underline{H(1)e^{j\frac{2\pi}{N}1.n}+H(1)^{*}e^{-j\frac{2\pi}{N}n}}+\underline{H(2)e^{j\frac{2\pi}{N}2.n}+H(2)^{*}e^{-j\frac{2\pi}{N}2n}}+.............\\ .....+\underline{H\left(\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n}+H\left(\frac{N-1}{2}\right)^{*}e^{-j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n}} \end{array}\right]$

$$h(n)=\frac{1}{N}\left[\begin{array}{l} H(0)+\underline{H(1)e^{j\frac{2\pi}{N}1.n}+\left(H(1)e^{j\frac{2\pi}{N}n}\right)^{*}}+\underline{H(2)e^{j\frac{2\pi}{N}2.n}+\left(H(2)e^{j\frac{2\pi}{N}2n}\right)^{*}}+.............\\ .....+\underline{H\left(\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n}+\left(H\left(\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n}\right)^{*}} \end{array}\right]$$

$$\therefore h(n)=\frac{1}{N}\left[H(0)+2\operatorname{Re}\left\{H(1)e^{j\frac{2\pi}{N}1.n}\right\}+2\operatorname{Re}\left\{H(2)e^{j\frac{2\pi}{N}2.n}\right\}+.............2\operatorname{Re}\left\{H\left(\frac{N-1}{2}\right)e^{j\frac{2\pi}{N}\left(\frac{N-1}{2}\right).n}\right\}\right]$$

$$\therefore h(n)=\frac{1}{N}\left[H(0)+2\sum_{k=1}^{\frac{N-1}{2}}\operatorname{Re}\left\{H(k)e^{j\frac{2\pi}{N}k.n}\right\}\right]$$ When N is odd

$$\therefore h(n)=\frac{1}{N}\left[H(0)+2\sum_{k=1}^{\frac{N}{2}-1}\operatorname{Re}\left\{H(k)e^{j\frac{2\pi}{N}k.n}\right\}\right]$$ When N is even